Note on the Theory of Topographically Forced Planetary Waves in the Atmosphere

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ABSTRACT—Estimates of the mean vertical motions at the earth's surface associated with airflow over topography are presented for Northern Hemisphere winter and summer conditions, and the implications of these estimates for the theory of mean stationary waves in the atmosphere are discussed.

1. INTRODUCTION

The vertical motions at the earth's surface associated with horizontal airflow over the mountain ranges are of importance both for their direct effect on local climate (e.g., adiabatic warming and cooling with accompanying cloud and precipitation formation) and for their indirect effect on planetary climate by the forcing of stationary wave systems (e.g., Charney and Eliassen 1949). This vertical motion is given by

$$w_s^{(M)} = \mathbf{V}_s \cdot \nabla h = u_s \frac{\partial h}{a \cos \phi \partial \lambda} + v_s \frac{\partial h}{a \partial \phi} \qquad (1)$$

where $\mathbf{V}$ is the vector horizontal wind of which u is the eastward component and v is the northward component, the subscript s denotes the value at the anemometer level, the superscript M indicates topographic forcing, ∇ is the horizontal del operator, h is the topographic height above sea level, a is the radius of the earth, λ is longitude, and ϕ is latitude. If we use a bar to denote a time average, we have

$$\overline{w}_s^{(M)} = \overline{\mathbf{V}}_s \cdot \nabla h. \qquad (2)$$

The field of mean vertical motion given by eq (2) can be converted to an equivalent field of mean surface "heating" due to adiabatic ascent and descent. Denoting this quantity by $\overline{q}_s^{(M)}$, we can write the relationship in the form

$$\overline{q}_s^{(M)} = -(c_p \Gamma_s \rho_s g) \overline{w}_s^{(M)} \qquad (3)$$

where Γ is the "static stability," $-(\partial T/\partial p - RT/pc_p)$, p is pressure, R is the gas constant, c_p is the specific heat at constant pressure, T is temperature, ρ is density, and g is the acceleration of gravity.

If we define an average around a latitude circle of a time-averaged variable, $\overline{\xi}$, by

$$\xi_0 \equiv \frac{1}{2\pi} \int_0^{2\pi} \overline{\xi} d\lambda$$

and the "standing eddy" departure from this zonal average by

$$\xi_1 \equiv \overline{\xi} - \xi_0,$$

we can expand eq (2) in the form

$$\overline{w}_s^{(M)} = u_{0s} \frac{\partial h}{a \cos \phi \partial \lambda} + \left[u_{1s} \frac{\partial h}{a \cos \phi \partial \lambda} + (v_0 + v_1)_s \frac{\partial h}{a \partial \phi} \right]. \qquad (4)$$

The terms in brackets in eq (4) have been neglected in the forcing function in all theoretical studies of linear response of the atmosphere to large-scale airflow over mountains. (See summary by Saltzman 1968.) In fact, the most detailed study of this kind made thus far (Sankar-Rao 1965) is based on the further approximation that u_{0s} is uniform with latitude, having the value observed for 45°N. It is worthwhile, therefore, to see to what extent this simplified form of the forced vertical motion at the surface approximates the more complete representation given by eq (2).

2. COMPUTATIONS

For the purpose of comparing the complete and the simplified forms of the forced vertical motion equation at the surface, we present an estimate of $\overline{w}_s^{(M)}$ for the Northern Hemisphere, computed for winter (October–March) and summer (April–September) wind conditions. The basic data used are the smoothed topographic heights of Berkofsky and Bertoni (1955) and the mean observed winds of Buch (1954). Strictly speaking, the winds should be those measured at the local station anemometer levels, but these winds are difficult to determine for the hemisphere. Hence, as a first approximation, we have used Buch's 850-mb observed winds and in a few regions of high mountains have used his 700-mb winds. The results are shown in figures 1 and 2 where we present the mapped values of $\overline{w}_s^{(M)}$ for winter and summer, respectively, in units of 10^{-2} cm/s. [One can observe a fairly close relationship between the positive regions of $\overline{w}_s^{(M)}$ and the field of mean precipitation (Trewartha 1968); this indicates that a good deal of the observed precipitation is connected with orographic uplifting.]
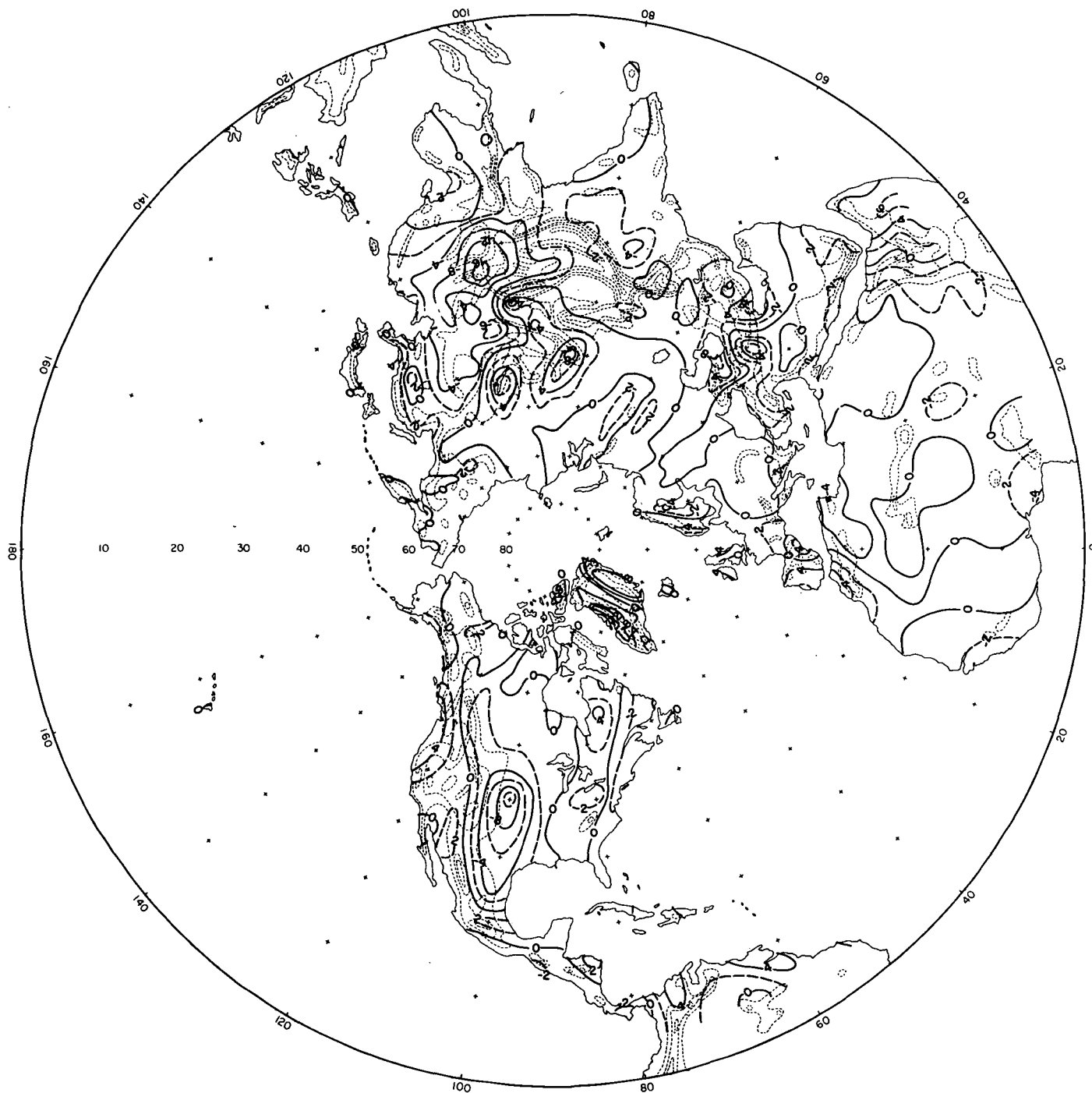
FIGURE 1.—Mean vertical motion (10^{-2} cm/s) at the surface, $\overline{w}_s^{(M)}$, associated with airflow over smoothed Northern Hemisphere topography in winter.

In figure 3 we show a comparison of the profile of $\overline{w}_s^{(M)}$ along 40°N, as derived from figures 1 and 2, with that obtained from the simplified form

$$\overline{w}_s^{(M)}* = u_{0s}\frac{\partial h}{a\,\cos\phi\partial\lambda}.\tag{5}$$

One can see that the amplitude of the forced vertical motion at the surface is much exaggerated by the approximate form [eq (5)], and in some places (e.g., 60°–75°E long.) the phase is seriously in error. This result is also repre-sentative of latitudes other than 40°N. Thus, one would expect that the amplitudes and phases of the stationary planetary waves deduced from models based on eq (5) are similarly in error. The discrepancy between $\overline{w}_s^{(M)}$ and $\overline{w}_s^{(M)}*$ is almost entirely due to the $\mathbf{V}_{1s}$-field, the term in-volving v_{0s} in eq (4) being negligibly small.

3. A SUGGESTED APPROACH TO THE STATIONARY WAVE THEORY

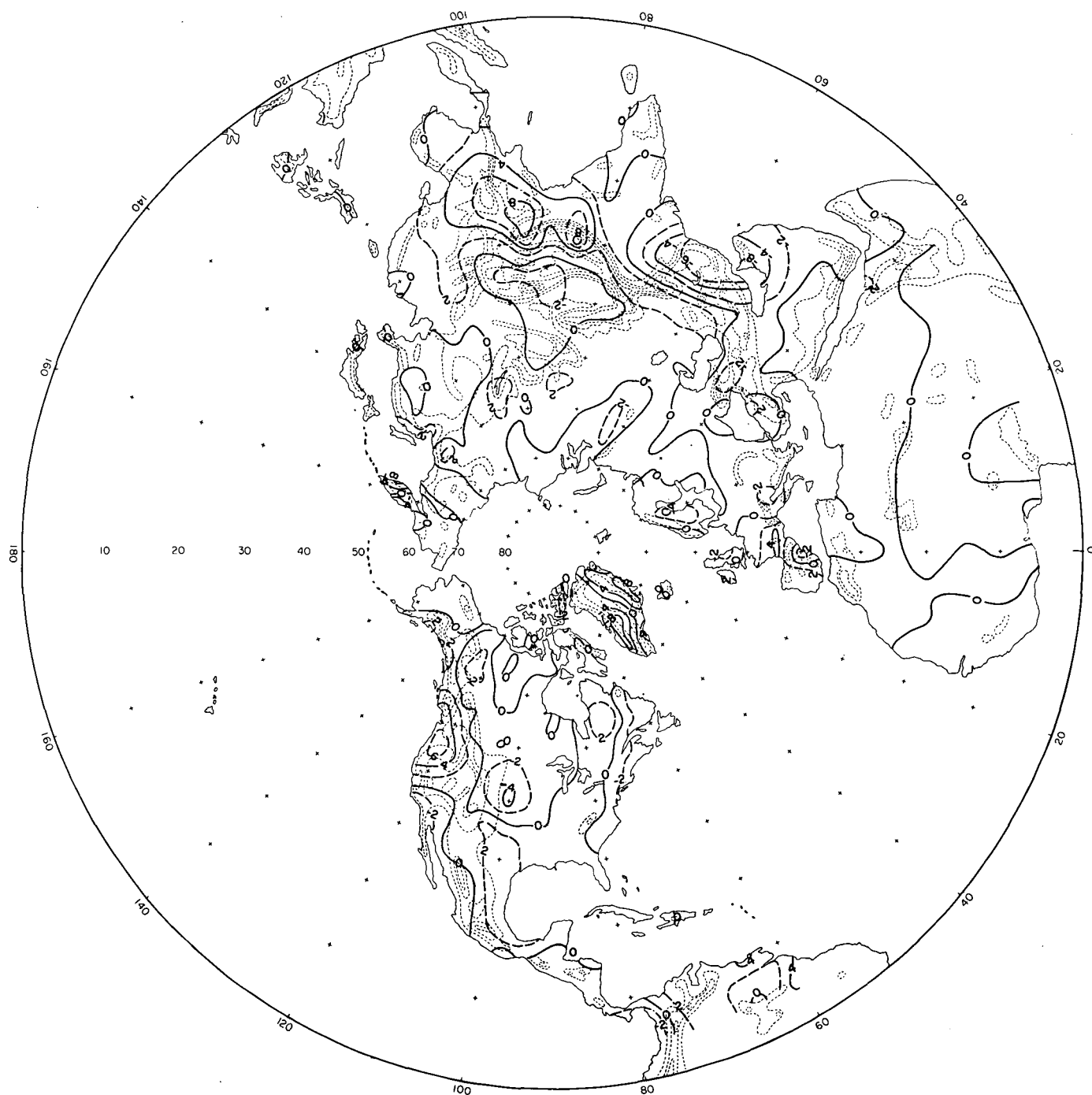In the linear theory of planetary standing waves in a

FIGURE 2.—Same as figure 1 for summer.

uniform zonal current, the forcing function includes thermal (i.e., monsoonal) effects and transient eddy effects as well as the topographic effects being discussed here (c.f., Smagorinsky 1953, Saltzman 1963). The effects of lateral deflection of currents *around* mountain barriers have generally been neglected. For this linear theory, the stationary wave solution can therefore be considered as a superposition of mean winds due to thermal and transient eddy forcing, $V_1^{(T)}$, and due to airflow over mountains, $V_1^{(M)}$; that is, $V_1 = V_1^{(T)} + V_1^{(M)}$. The existence of mountains also indirectly affects the thermal and transient eddy component through blocking effects on convective winds.

In light of the results in section 2, we suggest that the linear theory for V_1 be approached in two steps:

1. Given the zonally symmetric mean state (e.g., u_0), solve for V_1 due to the thermal forcing and the part of the topographic forcing represented by eq (5). The forcing due to transient eddy convergence of heat can be assumed to be included in the thermal forcing function, and the forcing due to the transient eddy convergence of momentum can be neglected (Saltzman 1963).

The thermal component of this solution, $V_1^{(T)}$, accounts for a good deal of the variance of the mean surface wind pattern (Sankar-Rao and Saltzman 1969, Sankar-Rao
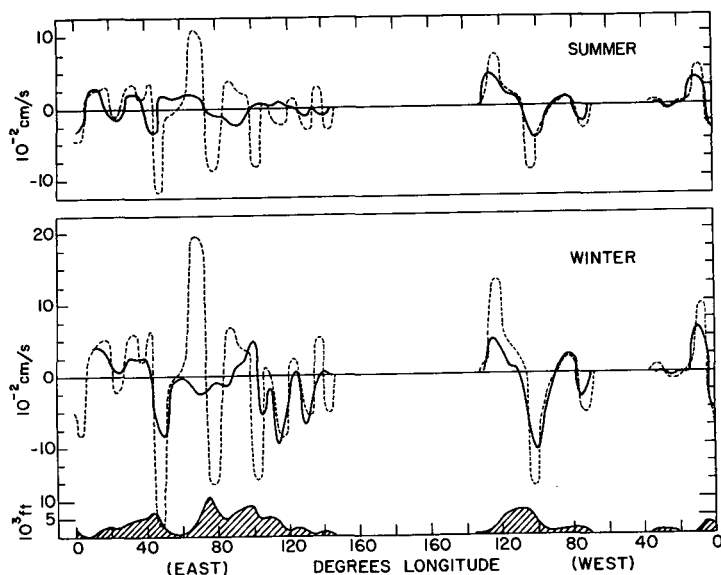
FIGURE 3.—Summer and winter profiles along 40°N of $\overline{w}_s^{(M)}$ (solid curve) computed from eq (2) and of $\overline{w}_s^{(M)}*$ (dashed curve) computed from eq (5). The topographic profile, $h(\lambda)$, is shown at the bottom.

(1970). On the other hand, the solution for $V_1^{(M)}$ corresponding to eq (5) shows maximum amplitude aloft with little amplitude near the surface (Sankar-Rao 1965). Thus the mean surface winds are generated primarily by the *thermal* part of the solution.

2. Using the surface winds, V_{1s}, generated by step (1), (i.e., by the thermal forcing, essentially) compute the response to the forced surface vertical motions,

$$w_{1s}^{(M)} = V_{1s} \cdot \nabla h.$$

This solution can now be added to that derived in step (1). Of course, this combined solution would still suffer from the inadequacies of the linear approximation, the use of a uniform zonal current, and the other approximations

commonly employed (Saltzman 1968), but we expect it to be better than that obtained by step (1) alone.

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